REPORT DOCUMENTATION PAGE

Form Approved
OBM No. 0704-0188

1. AGENCY USE ONLY *(Leave blank)*

2. REPORT DATE
July 1997

3. REPORT TYPE AND DATES COVERED
Proceedings

4. TITLE AND SUBTITLE
Reflection of a Short Narrow Light Pulse from a Scattering and Absorbing Ocean

5. FUNDING NUMBERS
Job Order No. 73664007
Program Element No. 062435N
Project No.
Task No.
Accession No.

6. AUTHOR(S)
Vladimir I. Haltrin

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Research Laboratory
Oceanography Division
Stennis Space Center, MS 39529-5004

8. PERFORMING ORGANIZATION REPORT NUMBER
NRL/PP/7331--97-0017

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Naval Research Laboratory
4555 Overlook Avenue, SW
Washington, DC 20375-5320

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES
Proceedings of the Third International Airborne Remote Sensing Conference and Exhibition, Volume II, 7-10 July 1997, Copenhagen, Denmark

12a. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution is unlimited.

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*
A very simple analytical expression for the shape of a pulse reflected from scattering and absorbing seawater is obtained. The resulting equation can be used for algorithms connected with the rapid assessment of optical water properties from remote platforms.

14. SUBJECT TERMS
optical properties, light pulses, emitter parameters, detector parameters, Gaussian, and scattered radiation

15. NUMBER OF PAGES
7

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
Unclassified	Unclassified	Unclassified	SAR

REFLECTION OF A SHORT NARROW LIGHT PULSE FROM A SCATTERING AND ABSORBING OCEAN*

Vladimir I. Haltrin

Naval Research Laboratory, Ocean Sciences Branch, Code 7331, Stennis Space Center, MS 39529-5004, USA
Phone: 601-688-4528, fax: 601-688-5379, e-mail: <haltrin@nrlssc.navy.mil>

ABSTRACT

A very simple analytical expression for the shape of a pulse reflected from scattering and absorbing seawater is obtained. The resulting equation can be used for algorithms connected with the rapid assessment of optical water properties from remote platforms.

1.0 INTRODUCTION

Even though there is a multitude of publications devoted to the propagation of a light pulse through a scattering and absorbing medium like seawater (see Refs. in Dolin and Levin, 1991), very few of them are practical enough to be used in real-time detection algorithms, mainly because of the complexity of the resulting equations. Instead, inadequate and oversimplified expressions are coded in many real detection programs. In this article an attempt is made to derive a very simple analytical expression for the spacial-temporal shape of a light pulse propagating in seawater. The resulting equation has a very simple form and depends parametrically on the characteristics of the emitter and detector, as well as the inherent optical properties of water.

1.1 Emitter and Detector Parameters

First, let us specify the parameters of an emitter and detector. For simplicity, a Lambert-Gaussian detector is assumed. Such a detector adequately emulates the majority of real detectors. The sensitivity of this detector is regarded as Lambertian, *i.e.*, it is independent of the angle of incidence of light. The sensitivity of the detector surface declines with the distance ρ from the center of the detector according to the Gaussian law:

$$T_D(\rho) = \frac{k_D}{4\rho^2} \exp\left(-\frac{\pi\rho^2}{4\rho_D^2}\right), \tag{1}$$

where ρ_D is the sensitivity radius which is defined by

$$\rho_D = \frac{2\pi}{k_D}\int_0^\infty T_D(\rho)\,\rho^2 d\rho, \tag{2}$$

and k_D is the detection efficiency of the receiver

$$k_D = 2\pi\int_0^\infty T_D(\rho)\,\rho\, d\rho. \tag{3}$$

We assume an emitter that is Gaussian both over the angle and over the distance from the center. It emits an infinitely short pulse represented by a temporal delta function. Such an assumption is mathematically convenient because the response from any arbitrary-shaped pulse can be calculated by mere convolution of the delta-shaped pulse response with the shape function of the real pulse (Morse and Feshbach, 1953). The energy density of the light pulse

* Presented at the Third International Airborne Remote Sensing Conference and Exhibition, 7-10 July 1997, Copenhagen, Denmark.

Proceedings of the Third
International Airborne Remote Sensing Conference and Exhibition
Development, Integration, Applications & Operations, Vol. II, Copenhagen, Denmark
Publication by Environmental Research Institute of Michigan (ERIM), ISSN 1076-7924, 7-10 July 1997.

$$\Delta_1(\mathbf{r},\mathbf{n}) = \frac{bf}{4\pi}\int_{\Omega_+} L_{p1}(\mathbf{r},\mathbf{n})\, p_-(-\mathbf{nn}')\,d\mathbf{n}' + \frac{b(1-f)}{4\pi}\int_{\Omega_+} L_{p2}(\mathbf{r},\mathbf{n})\, p_+(-\mathbf{nn}')\,d\mathbf{n}', \tag{21}$$

$$\Delta_2(\mathbf{r},\mathbf{n}) = \frac{bf}{4\pi}\int_{\Omega_+} L_{p2}(\mathbf{r},\mathbf{n})\, p_-(-\mathbf{nn}')\,d\mathbf{n}' + \frac{b(1-f)}{4\pi}\int_{\Omega_+} L_{p1}(\mathbf{r},\mathbf{n})\, p_+(-\mathbf{nn}')\,d\mathbf{n}'. \tag{22}$$

2.2 Small-Angle Approximation

Let us choose the phase function components p_+ and p_- in such a manner that their *tails* in the backward hemisphere are exponentially small, so that

$$|\Delta_1| << |Q_1| \text{ and } |\Delta_2| << |Q_2|. \tag{23}$$

The resulting phase function from Eqn.(10) with its parts satisfying the inequalities above still gives us a very satisfactory approximation for a realistic ocean phase function (Haltrin, 1984).

Let us solve Eqns. (18) in the small-angle approximation (Wells, 1982; Walker, 1987; Arnush, 1972; Dolin and Levin, 1991) with the phase functions described above. We should also make the following simplifications that are typical for the small-angle approximation:

$$\left.\begin{array}{c} \mathbf{n} = n_z + \mathbf{s},\quad n_z \cong \left(0,\ 0,\ 1-\mathbf{s}^2/2\right),\quad \mathbf{n}\nabla_r \cong \left(1-\frac{1}{2}\mathbf{s}^2\right)\frac{\partial}{\partial z} + \mathbf{s}\nabla_\rho,\quad p_\pm(\mathbf{nn}') \to p_\pm(|\mathbf{s}'|), \\ \mathbf{nn}' = 1-(\mathbf{n}-\mathbf{n}')^2/2,\quad \mathbf{n}-\mathbf{n}' = \mathbf{s}',\quad \mathbf{n}' = \mathbf{n}+(\mathbf{n}'-\mathbf{n}) = 1+(\mathbf{s}-\mathbf{s}'), \\ L_{pi}(\mathbf{r},\mathbf{n}) \to L_{pi}(z,\rho,\mathbf{s}),\quad L_{pi}(\mathbf{r},\mathbf{n}') \to L_{pi}(z,\rho,\mathbf{s}-\mathbf{s}'). \end{array}\right\} \tag{24}$$

Now we have the approximate system of equations for the Laplace transforms of radiances:

$$\left.\begin{array}{l} \left[\left(1-\frac{1}{2}\mathbf{s}^2\right)\frac{\partial}{\partial z} + \mathbf{s}\nabla_\rho + \varepsilon\right] L_{p1}(z,\rho,\mathbf{s}) = Q_1(z,\rho,\mathbf{s}), \\ \left[-\left(1-\frac{1}{2}\mathbf{s}^2\right)\frac{\partial}{\partial z} - \mathbf{s}\nabla_\rho + \varepsilon\right] L_{p2}(z,\rho,\mathbf{s}) = Q_2(z,\rho,\mathbf{s}). \end{array}\right\} \tag{25}$$

The Fourier transform of radiance in the plane that is orthogonal to the direction of the pulse propagation is:

$$L_{pi}(z,\rho,\mathbf{s}) = \iint d\mathbf{k}\,d\mathbf{q}\; F_{pi}(z,\mathbf{k},\mathbf{q})\, e^{-i\mathbf{k}\rho - i\mathbf{q}\mathbf{s}}, \tag{26}$$

$$F_{pi}(z,\mathbf{k},\mathbf{q}) = \frac{1}{(2\pi)^4}\iint d\rho\, ds\; L_{pi}(z,\rho,\mathbf{s})\, e^{i\mathbf{k}\rho + i\mathbf{q}\mathbf{s}}. \tag{27}$$

Now we have the following system of equations for the Laplace-Fourier transforms of the forward and backward radiances:

$$\left.\begin{array}{l} \left[\left(1-\frac{1}{2}\mathbf{s}^2\right)\frac{\partial}{\partial z} - \mathbf{k}\frac{\partial}{\partial \mathbf{q}} + \varepsilon\right] F_{p1}(z,\mathbf{k},\mathbf{q}) = V_+(\mathbf{q})\,F_{p1}(z,\mathbf{k},\mathbf{q}) + V_-(\mathbf{q})\,F_{p2}(z,\mathbf{k},\mathbf{q}), \\ \left[-\left(1-\frac{1}{2}\mathbf{s}^2\right)\frac{\partial}{\partial z} + \mathbf{k}\frac{\partial}{\partial \mathbf{q}} + \varepsilon\right] F_{p2}(z,\mathbf{k},\mathbf{q}) = V_+(\mathbf{q})\,F_{p2}(z,\mathbf{k},\mathbf{q}) + V_-(\mathbf{q})\,F_{p1}(z,\mathbf{k},\mathbf{q}), \end{array}\right\} \tag{28}$$

where

$$V_\pm(\mathbf{q}) = \frac{b(1-f)}{4\pi}\int p_\pm(|\mathbf{s}|)\, e^{i\mathbf{q}\mathbf{s}}\, ds = \frac{b(1-f)}{2}\int_0^{\theta_{max}} J_0(q\theta)\, p_\pm(\theta)\, d\theta, \tag{29}$$

and J_0 is the zero-order Bessel function. Retaining only terms proportional to $\mathbf{q}^2$, we get:

$$\left.\begin{aligned} V_+(\mathbf{q}) &\cong b(1-f) - b\langle\theta^2\rangle \mathbf{q}^2/4, \\ V_-(\mathbf{q}) &\cong b\,f - b\langle(\pi-\theta)^2\rangle \mathbf{q}^2/4, \end{aligned}\right\} \tag{30}$$

where the angular brackets $\langle ... \rangle$ denote averaging over the phase function given by Eqn. (10) according to the rule:

$$\langle x(\mu)\rangle = \frac{1}{2}\int_{-1}^{1} p(\mu)\,x(\mu)\,d\mu \ . \tag{31}$$

Equations (28) for the light pulse components should satisfy the following boundary condition:

$$F_{p1}(z,\mathbf{k},\mathbf{q})\Big|_{z=0} = F_{p0}(\mathbf{k},\mathbf{q}) = P_0 \exp\left(-\frac{\rho_E^2\mathbf{k}^2}{\pi} - \frac{D_\theta\,\mathbf{q}^2}{4}\right). \tag{32}$$

Now we can estimate the terms in the left part of Eqn. (30) which are proportional to $\mathbf{s}^2$:

$$\frac{1}{2}\mathbf{s}^2\frac{\partial}{\partial z}L_i \sim \langle\theta^2\rangle c\,L_i. \tag{33}$$

At the same time our corrections due to the phase function in Eqn (27) have the following order of magnitude:

$$\frac{b}{4}\langle\theta_\pm^2\rangle q^2\,L_i \sim b\langle\theta^2\rangle\frac{1}{\langle\theta^2\rangle}L_i \sim b\,L_i. \tag{34}$$

So, if the condition $\langle\theta^2\rangle << \omega_0$ holds, all terms which are proportional to s^2 in the left part of Eqn. (28) may be neglected. With this in mind, Eqns. (24) acquire the following form:

$$\left.\begin{aligned} &\left(\frac{\partial}{\partial z} - \mathbf{k}\frac{\partial}{\partial \mathbf{q}} + \alpha + \beta_1\mathbf{q}^2\right) F_{p1}(z,\mathbf{k},\mathbf{q}) - \ (bf - \beta_2\,\mathbf{q}^2)\ F_{p2}(z,\mathbf{k},\mathbf{q}) = 0\,, \\ -(bf - \beta_2\,\mathbf{q}^2)\ &F_{p1}(z,\mathbf{k},\mathbf{q}) + \left(-\frac{\partial}{\partial z} + \mathbf{k}\frac{\partial}{\partial \mathbf{q}} + \alpha + \beta_1\mathbf{q}^2\right) F_{p2}(z,\mathbf{k},\mathbf{q}) = 0\,, \end{aligned}\right\} \tag{35}$$

where $\alpha = a + bf + p/v$, $\beta_1 = b < \theta^2 > /4$ and $\beta_2 = b < (\pi-\theta)^2 > /4$, and the boundary condition for Eqns. (35) is given by Eqn. (32).

Next, let us represent the downward pulse radiance as a sum of the unscattered part F_{p1}^Q (the source) and the scattered part F_{p1}^s:

$$F_{p1}(z,\mathbf{k},\mathbf{q}) = F_{p1}^Q(z,\mathbf{k},\mathbf{q}) + F_{p1}^s(z,\mathbf{k},\mathbf{q}). \tag{36}$$

The backward pulse radiance consists only of scattered radiation $F_{p2}(z,\mathbf{k},\mathbf{q}) = F_{p2}^s(z,\mathbf{k},\mathbf{q})$. The unscattered forward pulse radiance F_{p1}^Q satisfies the following propagation equation:

$$\left(\frac{\partial}{\partial z} - \mathbf{k}\frac{\partial}{\partial \mathbf{q}} + \alpha + \beta_1\mathbf{q}^2\right) F_{p1}^Q(z,\mathbf{k},\mathbf{q}) = 0\,, \tag{37}$$

with the same condition on the boundary given by Eqn. (29). The solution to Eqn. (37) is given by the expression:

$$F_{p1}^{Q}(z,\mathbf{k},\mathbf{q}) = F_{p}^{0}(\mathbf{k},\mathbf{q}+\mathbf{k}z)\exp\left[-\alpha z-\beta_1\int_0^z(\mathbf{q}+\mathbf{k}\eta)^2\,d\eta\right]. \tag{38}$$

This expression can also be represented by the following analytical formula:

$$F_{p1}^{Q}(z,\mathbf{k},\mathbf{q}) = P_0\exp\left[-\frac{\rho_E^2\mathbf{k}^2}{\pi}-\frac{D_\theta\mathbf{q}^2}{4}-\left(\alpha+\frac{D_\theta}{2}\mathbf{kq}+\beta_1\mathbf{q}^2\right)z-\left(\frac{D_\theta}{4}\mathbf{k}^2+\beta_1\mathbf{kq}\right)z^2-\beta_1\frac{\mathbf{k}^2}{3}z^3\right]. \tag{39}$$

The scattered parts of the forward and backward radiances of the pulse are satisfied by the following equations:

$$\left.\begin{aligned}\left(-\frac{\partial}{\partial z}+\mathbf{k}\frac{\partial}{\partial\mathbf{q}}+\alpha+\beta_1\mathbf{q}^2\right)F_{p2}^{s}(z,\mathbf{k},\mathbf{q}) &= (bf-\beta_2\mathbf{q}^2)\,F_{p1}^{Q}(z,\mathbf{k},\mathbf{q}),\\ \left(\frac{\partial}{\partial z}-\mathbf{k}\frac{\partial}{\partial\mathbf{q}}+\alpha+\beta_1\mathbf{q}^2\right)F_{p1}^{s}(z,\mathbf{k},\mathbf{q}) &= (bf-\beta_2\mathbf{q}^2)\,F_{p2}^{s}(z,\mathbf{k},\mathbf{q}),\end{aligned}\right\} \tag{40}$$

with the boundary conditions

$$F_{p1}^{s}(z,\mathbf{k},\mathbf{q})\Big|_{z=0}=0,\quad \lim_{z\to\infty}F_{pi}^{s}(z,\mathbf{k},\mathbf{q})=0,\quad i=1,2. \tag{41}$$

The next step, according to the method given by Both (1929), Snyder (1949) and Romanova (1968), involves the following substitutions:

$$\mathbf{q}=\mathbf{g}-\mathbf{k}z,\quad F_{pi}^{s}(z,\mathbf{k},\mathbf{q}) = F_{pi}^{s}(z,\mathbf{k},\mathbf{g}-\mathbf{k}z)\equiv\Phi_{pi}^{s}(z,\mathbf{k},\mathbf{g}), \tag{42}$$

which convert Eqns. (40) into the following one-dimensional system of two equations:

$$\left.\begin{aligned}\left[-\frac{d}{dz}+\alpha+\beta_1(\mathbf{g}-\mathbf{k}z)^2\right]\Phi_{p2}^{s}(z,\mathbf{k},\mathbf{g}) &= \left[bf-\beta_2\,(\mathbf{g}-\mathbf{k}z)^2\right]F_{p1}^{Q}(z,\mathbf{k},\mathbf{g}-\mathbf{k}z),\\ \left[\frac{d}{dz}+\alpha+\beta_1\,(\mathbf{g}-\mathbf{k}z)^2\right]\Phi_{p1}^{s}(z,\mathbf{k},\mathbf{g}) &= \left[bf-\beta_2\,(\mathbf{g}-\mathbf{k}z)^2\right]\Phi_{p2}^{s}(z,\mathbf{k},\mathbf{g}).\end{aligned}\right\} \tag{43}$$

The solutions to Eqns. (43) can be found with the help of the following two (forward, G_+, and backward, G_-) Green functions (Vladimirov, 1971):

$$G_\pm(z,\mathbf{k},\mathbf{g}) = H(\pm z)\exp\left[\mp\alpha z\mp\beta_1\int_0^z(\mathbf{g}-\mathbf{k}\eta)^2\,d\eta\right], \tag{44}$$

which are solutions to the equations:

$$\left[\pm\frac{d}{dz}+\alpha+\beta_1(\mathbf{g}-\mathbf{k}z)^2\right]G_\pm(z,\mathbf{k},\mathbf{g}) = \delta(z), \tag{45}$$

where $H(z)$ ($H=1$, for $z>0$, $H=0$, for $z\le 0$) is a Heavyside or step function (Morse and Feshbach, 1953).
The final solutions to Eqns. (43) are:

$$\Phi_{p2}^{s}(z,\mathbf{k},\mathbf{g}) = \int G_-(z-\xi,\mathbf{k},\mathbf{g})\left[bf-\beta_2(\mathbf{g}-\mathbf{k}\xi)^2\right]F_{p1}^{Q}(\xi,\mathbf{k},\mathbf{g}-\mathbf{k}\xi)\,d\xi, \tag{46}$$

Proceedings of the Third
International Airborne Remote Sensing Conference and Exhibition
Development, Integration, Applications & Operations, Vol. II, Copenhagen, Denmark
Publication by Environmental Research Institute of Michigan (ERIM), ISSN 1076-7924, 7-10 July 1997.

$$\Phi^s_{p1}(z,\mathbf{k},\mathbf{g}) = \int G_+(z-\xi,\mathbf{k},\mathbf{g})\left[bf - \beta_2(\mathbf{g}-\mathbf{k}\xi)^2\right]\Phi^s_{p2}(\xi,\mathbf{k},\mathbf{g})\big)d\xi + C_1(\mathbf{k},\mathbf{g})\exp\left[-\alpha z - \beta_1\int_0^z(\mathbf{g}-\mathbf{k}\eta)^2 d\eta\right], \quad (47)$$

where $C_1(\mathbf{k},\mathbf{g})$ is an arbitrary function that is determined by the boundary condition.

After simplification, we obtain the following expressions for F_{pi}:

$$F_{p1}(z,\mathbf{k},\mathbf{q}) = F^0_p(\mathbf{k},\mathbf{q}+\mathbf{k}z)\exp[-\alpha z - \beta_1\int_0^z(\mathbf{q}+\mathbf{k}\eta)^2 d\eta]\left\{1+\int_0^z d\xi\left[bf-\beta_2(\mathbf{q}-\mathbf{k}\xi)^2\right]\times\right.$$
$$\left.\times\exp[-\beta_1\int_{\xi-z}^{z}(\mathbf{q}+\mathbf{k}\eta)^2 d\eta]\int_{-\infty}^{0} d\varsigma\left[bf-\beta_2(\mathbf{q}-\mathbf{k}\xi+\mathbf{k}\varsigma)^2\right]\exp[2\alpha\varsigma-\beta_1\int_{\varsigma}^{\xi-\varsigma}(\mathbf{q}-\mathbf{k}\eta)^2 d\eta]\right\}, \quad (48)$$

$$F_{p2}(z,\mathbf{k},\mathbf{q}) = F^0_p(\mathbf{k},\mathbf{q}+\mathbf{k}z)e^{-\alpha_F z}\int_{-\infty}^{0} d\xi\left[bf-\beta_2(\mathbf{q}-\mathbf{k}z+\mathbf{k}\xi)^2\right]\exp[2\alpha\xi-\beta_1\int_{\xi}^{z-\xi}(\mathbf{q}-\mathbf{k}\eta)^2 d\eta]\ . \quad (49)$$

2.3 Detector Response to the Infinitely Short Laser Pulse Reflected from Water

After the integration of the received radiances over the sensitive area of the detector, the relative (normalized by the pulse power P_0) response of the detector placed at the depth z will be

$$\eta_1(z,w) = \frac{\delta(w)\,v\,e^{-\alpha(w+z)}}{1+\left(\frac{\rho_0}{\rho_A}\right)^2\left[1+\frac{4D_1\sigma_0}{3s_0^2}\right]\frac{s_0 z}{\rho_A}} + \int_0^z \frac{\theta(w)(\beta\rho_A)^2\,v\,e^{-\alpha(w+z)}\,d\xi}{2a_1+2D_2\left(2\xi^2-w\xi\right)+D_1\sigma_0\left(\frac{4\xi^3}{3}+2w\xi^2-w^2\xi\right)}, \quad (50)$$

where

$$a_1 = \rho_A^2+\rho_0^2+s_0^2 z^2+D_2 w^2+\frac{D_1\sigma_0}{3}(4z^3+w^3),\ \ w = vt-z, \quad (51)$$

$$\left.\begin{aligned}\eta_2(z,w) &= \frac{\beta v}{2D_w}\theta(w)\exp[-\alpha(w+z)],\\ D_w &= 1+\left(\frac{\rho_0}{\rho_A}\right)^2+D_2\left(1+\frac{D_1\sigma_0 w}{3D_2}\right)\left(\frac{w}{\rho_A}\right)^2+\left(1+\frac{4D_1\sigma_0}{3s_0^2}z\right)\left(\frac{s_0 z}{\rho_A}\right).\end{aligned}\right\} \quad (52)$$

At $z=0$, the detector response to the reflected pulse will be:

$$\eta_2(\tau) = \frac{a_0\,\theta(\tau)\,e^{-\tau}}{1+a_2\tau^2+a_3\tau^3}, \quad (53)$$

where

$$\tau = t/t_0\ ,\ t_0 = (\alpha v)^{-1}\ ,\ a_0 = \frac{8\pi^7\omega_0 f}{[1-\omega_0(1-f)]\left[1+(\rho_E/\rho_D)^2\right]} \quad (54)$$

$$a_2 = \frac{\pi D_2}{8c^2\left(\rho_E^2+\rho_D^2\right)[1-\omega_0(1-f)]^2},\ \ a_3 = \frac{\pi\omega_0(1-f)D_1}{24c^2\left(\rho_E^2+\rho_D^2\right)[1-\omega_0(1-f)]^3}, \quad (55)$$

where *f* is the weight coefficient in Eqn. (10).

The values of η_2 calculated according to Eqns. (53)-(55) were compared with the results computed with the Monte Carlo code by Kattawar (1992). The discrepancies between the natural logarithms of the η_2 computed by both these methods for inherent optical properties and phase functions by Petzold (1972) do not exceed 15%.

3.0 CONCLUSIONS

Relatively simple analytical equations derived in a small-angle scattering approximation have been obtained for the infinitely short light pulse reflected by seawater. These equations depend on the inherent optical properties of seawater, as well as the parameters of the pulse and receiver. They can be transformed into equations for an arbitrarily-shaped pulse by a simple convolution procedure. The logarithmic precision of these equations is estimated to be in the range of 15%. The results of this paper can be used for algorithms connected with the rapid assessment of water properties from remote airborne and shipborne platforms.

4.0 ACKNOWLEDGMENT

The author is grateful to Dr. George W. Kattawar for the opportunity to use his DAIREBORNENEW code and to Walton E. McBride III and Elena V. Haltrin for useful help. This work was supported at the Naval Research Laboratory through the Littoral Optical Environment (LOE 6640-07) and Optical Oceanography (OO 73-5051-07) programs. This article represents NRL contribution NRL/PP/7331–97–0017.

5.0 REFERENCES

D. Arnush, "Underwater Light-Beam Propagation in the Small-Angle Approximation", *J. Opt. Soc. Am.*, Vol. 62, pp. 1109-1111, 1972.

W. Both, "Die Streuabsorption der Electronenstrahlen," (in German) *Z. Phys.*, 54, pp. 167-178, 1929.

L. S. Dolin, and I. M. Levin, *Theory of Underwater Vision: Reference Book* (in Russian), Gidrometeoizdat, Leningrad, pp. 228, 1991.

V. I. Haltrin (a.k.a. V. I. Khalturin), "Propagation of Light in Sea Depth" , Ch. 2 in *Optical Remote Sensing of the Sea and the Influence of the Atmosphere* (in Russian), eds. V. A. Urdenko and G. Zimmermann, GDR Academy of Sciences Institute for Space Research, Moscow-Berlin-Sevastopol, pp. 20-62, 1985.

V. I. Haltrin, "Exact Solution of the Characteristic Equation for Transfer in the Anisotropically Scattering and Absorbing Medium", *Appl. Optics*, 27(3), p.599-602, 1988.

V. I. Haltrin, "Theoretical and empirical phase functions for Monte Carlo calculations of light scattering in seawater," in *Proceedings of the Fourth International Conference Remote Sensing for Marine and Coastal Environments: Technology and Applications*, Vol. I, Publ. by Envir. Research Instit. of Michigan (ERIM), Ann Arbor, MI, USA, p. 509-518, 1997.

G. W. Kattawar, "FORTRAN program DAIREBORNENEW", (private communication), Department of Physics, Texas A&M University, College Station, TX, 1992.

P. M. Morse, and H. Feshbach, *Methods of Theoretical Physics*, Vols.1 and 2, McGraw-Hill Book Co., New York-Toronto-London, pp. 1978, 1953.

L. M. Romanova, "The Light Field in deep Layers of a Turbid Medium Illuminated by a Narrow Beam", *Izv. USSR AS, Atm. Ocean Phys.*, Vol.4, p. 679, 1968.

Petzold, T. J., *Volume Scattering Functions for Selected Ocean Waters*, SIO Ref. 72-78, Scripps Institute of Oceanography, Visibility Laboratory, San Diego, CA, 1972, 79 p.

H. S. Snyder, W. T. Scott, "Multiple Scattering of Fast Charged Particles", *Phys. Rev.*, 76(2), pp. 220-225, 1949.

V. S. Vladimirov, *Equation of Mathematical Physics*, M. Dekker, New York, N.Y., pp. 418, 1971.

P. L. Walker, "Beam Propagation through Slab Scattering Media in the Small-Angle Approximation", *Appl. Optics*, 26(3), pp. 524-528, 1987.

W. H. Wells, "Theory of Small-Angle Scattering", *Optics of the Sea, AGARD Lect. Ser 61*, Ch.3.3., 1973.